

“On the Dielectric Constant of Liquid Oxygen and Liquid Air.” By J. A. FLEMING, M.A., D.Sc., F.R.S., Professor of Electrical Engineering in University College, London, and JAMES DEWAR, M.A., LL.D., F.R.S., Fullerian Professor of Chemistry in the Royal Institution, &c. Received December 8,—Read December 17, 1896.

The exceedingly high insulating properties of liquid oxygen and liquid air indicate that these bodies are dielectrics, and possess a dielectric constant or specific inductive capacity which it is necessary to determine. We have, therefore, lately made some measurements which have enabled us to assign a number representing, in all probability, a close approximation to these constants.

The remarkable non-conducting quality of these liquid gases for electricity enabled us to employ a method which, generally speaking, is not applicable to liquids other than those of very high specific resistance, or insulating power.

The method used by us in these experiments consists in the employment of a small condenser composed of metal plates which can be plunged beneath the surface of the liquid gas, and the capacity of this condenser measured when the dielectric between the plates is first gaseous air at ordinary temperature and pressure, and is next replaced by the liquid oxygen or liquid air. In order to determine the capacity of this condenser, which is necessarily small and of the order of 0·001 microfarad, we adopted the well-known device of charging the small condenser with a high potential (100 volts) and then discharging it into a much larger, well insulated mica condenser, having a capacity of about 0·5 microfarad. This process was repeated ten times, and the larger condenser was then discharged through a standardised ballistic galvanometer. A specially constructed and highly insulated key was employed to charge the small condenser by means of a battery of fifty small lithanode secondary cells; and then to discharge it into the larger condenser. The success of this method depends entirely on the absence of sensible leakage in the condensers, and it is essential to show that the small condenser loses no sensible portion of its charge by leakage or conduction during the interval which elapses between disconnecting it from the battery and connecting it to the large condenser, which acts as a reservoir.

In these experiments the small condenser consisted of seventeen plates of carefully flattened aluminium, about 1 mm. in thickness; each plate being 5 cm. wide by 15 cm. long. In order to separate the plates, small distance pieces of crown glass were employed,

each fragment being about 3 mm. square and 1 mm. thick. Four of these fragments were affixed to each metal plate with a touch of shellac at the four corners and one fixed in the middle. The seventeen plates were then piled one on the other, the glass fragments acting as separators, and the alternate plates were connected together by wires soldered to each series. A metal clamp kept all the plates in position. The condenser so formed consisted of seventeen plates, eight being the positive, and nine the negative surfaces. The glass distance pieces had a total surface of very nearly 1 per cent. of the total opposed surface of the plates. The condenser so formed had a capacity of 0.001031 of a microfarad when gaseous air at 15° C. and normal pressure formed the dielectric.

If such a condenser having a capacity C' is charged to a potential V and then discharged n times in succession into a larger reservoir condenser of capacity C , it is easy to show that at the end of the n successive charges the quantity Q contained in the large condenser is given by the series

$$Q = C'V(m + m^2 + m^3 \dots m^n),$$

where

$$m = \frac{C}{C + C'}.$$

Hence we have

$$Q = C'V \frac{m}{1-m} (1 - m^n).$$

The capacity C' of the small aluminium condenser may be considered to be made up of two parts; a part which is changed when liquid oxygen is substituted for gaseous oxygen or air on immersing the condenser, and which thereby becomes increased. If K is the dielectric constant of liquid oxygen, referred to that of gaseous oxygen at -182° C. as unity; and if c is the capacity of this variable part of the condenser when the dielectric is gaseous oxygen, then Kc is its capacity when liquid oxygen is substituted for the gaseous oxygen at the same temperature.

In the next place there is a small part of the whole capacity due to the glass separators. These, as a whole, have a surface very nearly equal to 1 per cent. of the whole surface of the metal plates, and a dielectric constant, as shown below, when cooled to -182° C., of 5.0. Hence it follows that that part of the whole capacity of the condenser which is due to the glass separators, may be represented very nearly by $5c/100$.

This part of the capacity remains practically constant whether the condenser is lifted out of the liquid oxygen into the cold gaseous oxygen lying above it, and which is at nearly the same temperature, or put into it, as long as the condenser is very nearly at the same temperature in the two conditions.

Hence, when the small condenser is under the surface of liquid oxygen its capacity C' , as a whole, is

$$Kc + 0.05c,$$

and the whole quantity of electricity, Q , given up to the reservoir condenser after n charges of the small one, charged to potential V , have been put into it, is

$$Q = Vc(K + 0.05) \frac{m}{1-m} (1-m^n) \\ = Vc(K + 0.05)M,$$

where $m = \frac{C}{C + (K + 0.05)c}$ and $M = \frac{m}{1-m} (1-m^n).$

Again, when the small condenser is lifted out of the liquid oxygen into the gaseous oxygen lying on the surface, its capacity becomes $c + 0.05c = 1.05c$, and the whole quantity Q' stored up in the reservoir condenser, after n charges at a potential V , is

$$Q' = Vc(1.05) \frac{m'}{1-m'} (1-m'^n) \\ = Vc(1.05)M',$$

where $m' = \frac{C}{C + 1.05c}$ and $M' = \frac{m'}{1-m'} (1-m'^n).$

If in each case the reservoir condenser is discharged through a ballistic galvanometer, the "throw" or elongation of which is proportional to the quantity of electricity sent through it, and if θ and θ' are the throws produced by the quantities Q and Q' , we have

$$\frac{\theta}{\theta'} = \frac{Q}{Q'} = \frac{K + 0.05}{1.05} \frac{M}{M'}.$$

The ratio θ/θ' is given from the observations.

To solve this equation completely and determine K would be difficult, since the quantity M is a somewhat complicated function of K .

We know, however, that the ratio of M/M' cannot be very far from unity. A rough experiment had shown that K was a number in the neighbourhood of 1.5, and a calculation shows that when ten discharges of the small condenser are made in each case into the large condenser, and if the large condenser has a capacity of 0.5 microfarad, and the small one a capacity of nearly 0.001 microfarad, that the ratio $M/M' = 1030/1019$ nearly. Hence M/M' comes in as a correcting factor of about 1 per cent. in value.

Before relying on the above method, it was necessary to prove that the loss of charge of the small condenser was negligible during the time elapsing between the end of the charge and the end of the discharge of the small condenser.

We found on trial that although the small condenser had a capacity of only 0.001031 microfarad, it held its charge when charged with 100 volts, and placed beneath the surface of liquid air in the most extraordinary way. The test for insulation was as follows:—

The small condenser was charged with 100 volts, and discharged through the galvanometer instantly. The galvanometer throw was 95 scale divisions.

The small condenser was then charged and allowed to stand ten minutes insulated. It was then discharged through the galvanometer, and the throw was 90 scale divisions. In like manner it was charged and insulated for forty-seven minutes, and the throw was then 80 scale divisions.

The above figures show that the charge of the small insulated condenser decreased only by about 15 per cent. in three-quarters of an hour when placed beneath liquid air, and hence the loss of charge in one-tenth of a second was quite inappreciable.*

The same remarkable insulation is found when the small condenser is held in the cold gaseous oxygen lying above the liquid oxygen. The low temperature of -182°C . prevents any sensible leakage across the glass distance pieces, and also increases the specific resistance of the glass itself.

As a further instance of the very high insulating power of liquid air, we may mention that we charged the small condenser when immersed in liquid air with a Wimshurst electrical machine, and, after insulating the condenser and waiting a few moments, closed the terminals of the condenser by a wire. A small spark was seen at the contacts. We thus constructed a little Leyden jar, the dielectric of which was liquid air, and the coatings the aluminium plates. This liquid Leyden jar held its charge perfectly.

Having satisfied ourselves in this manner that the condenser when immersed in liquid air would lose no sensible portion of its charge during the fraction (about one-tenth) of a second in which the charge and discharge key was moving between its contacts, we proceeded to experiment in the following manner. The condenser was placed in a very large vacuum vessel, holding about two litres of liquid oxygen, and it was charged as described, and discharged into a very good mica condenser, made by Dr. Muirhead, which had an exceedingly high insulation. The process of charging and discharging ten times occupied, perhaps, two seconds.

* These figures do not of course measure the electrical resistance of the liquid oxygen alone. They show, however, that the immersion of the condenser in liquid oxygen enormously decreased or entirely destroyed any surface leakage over the small glass separators, and, as we have found by an independent examination, increased the resistivity of the glass itself. The specific resistance of liquid oxygen itself is exceedingly high.

The resultant charge having been measured on the ballistic galvanometer, the condenser was lifted out into the cold gaseous oxygen lying on the surface of the liquid oxygen, and before the condenser had time to alter its form by rising in temperature, the same process was repeated with the dielectric changed to gaseous oxygen at -182°C .

The following Table I shows the observed ballistic throws, all reduced to their equivalents at one common charging pressure of 100 volts:—

Table I.—Observations to Determine the Dielectric Constant of Liquid Oxygen.

	Potential to which the condenser was charged in volts.	Ballistic throw in cm., corresponding to 10 charges of the small condenser.	Ballistic throw, reduced to correspond to 10 charges of the small condenser at 100 volts.
Exp. I.—Condenser at ordinary temperature, 15°C .	103.3	7.7	7.45
	103.2	7.75	7.51
	103.2	7.75	7.51
Exp. II.—Condenser in liquid oxygen at -182°C .	103.15	11.3	10.96
	103.1	11.25	10.91
	103.1	11.27	10.93
	103.0	11.27	10.94
Exp. III.—Condenser in cold oxygen gas above the liquid oxygen at -182°C .	101.3	7.65	7.55
	101.2	7.6	7.51
	101.2	7.6	7.51
	101.2	7.58	7.49
	101.2	7.56	7.51
Exp. IV.—Condenser in liquid oxygen.	101.3	10.9	10.77
	101.2	10.85	10.72
			} bad.
Exp. V.—Condenser in cold oxygen gas above the liquid oxygen.	101.3	7.60	7.50
	101.3	7.60	7.50
	101.3	7.58	7.48
	101.3	7.57	7.47
Exp. VI.—Condenser in liquid oxygen.	101.4	11.1	10.95
	101.3	11.0	10.86
	101.3	10.95	10.81
	101.3	11.0	10.86

Mean ballistic throw in gaseous oxygen = $7.502 = \theta'$. Mean ballistic throw in liquid oxygen = $10.903 = \theta$.

It will be seen that the mean galvanometer throw, when the condenser was immersed in liquid oxygen, was 10·903 centims., and the mean throw, when raised into the gaseous oxygen, was 7·146 centims.

One matter which we felt it important to examine, was whether there was any correction needed for the change in the dielectric constant of the glass separators with temperature.

Since these glass separators had a total surface of nearly 1 per cent. of the area of the metal plates, the condenser may be regarded as consisting of two condensers joined in parallel, one consisting of a glass dielectric condenser having an effective surface of 1, and the other a condenser having a liquid or gaseous oxygen dielectric having an effective area of 99. In the course of these experiments we have therefore examined the effect of low temperature upon the dielectric constants of glass, paraffined paper, and mica. We find that on cooling these bodies to -182°C . they experience a marked reduction in dielectric constant. The dielectric constant of a certain specimen of crown glass was reduced by 21·4 per cent. by cooling to the temperature of liquid air or to -185°C . The dielectric constant of paraffined paper was reduced by 28·4 per cent. under the same circumstances.* We are engaged in a systematic examination of the influence of very low temperatures on the dielectric constants and specific resistances of the principal dielectric bodies. The crown glass used as separators in the construction of our small condenser had a specific inductive capacity of about 6·0 at ordinary temperature, and this at the low temperature would be reduced to nearly 5·0. Hence in estimating the capacity of the condenser, as constructed, there comes in as we have seen a correction from the presence of the glass. We selected glass in the first instance rather than ebonite or sulphur, as we thought it probable we should use the same condenser in determining other dielectric constants, and we wished to construct the separators of a material which was very rigid and not easily acted upon by oils or other liquids.

Taking the formula above given, we can deduce from the observed results the required constant, for, we have

$$\frac{\theta}{\theta'} = \frac{K + 0\cdot05}{1\cdot05} \frac{1030}{1019},$$

and hence substituting for $\frac{\theta}{\theta'}$ the observed ratio $\frac{10\cdot903}{7\cdot502}$, we find

$$K = 1\cdot491$$

* By another method we have found that for the glass of a glass test-tube the dielectric constant was decreased 22·2 per cent. by cooling to the temperature of liquid air. Under the same circumstances a certain specimen of mica decreased only 3·01 per cent. in dielectric constant.

as the dielectric constant of liquid oxygen referred to that of the overlying gaseous oxygen at -182° C. as unity. Since the aluminium condenser is at the same temperature when the two measurements are made, no correction is necessary for any change of form of the condenser.

To determine the dielectric constant of liquid oxygen in terms of that of a vacuum taken as unity, we require to know the dielectric constant of the gaseous oxygen lying on the surface of the liquid oxygen referred to the same unit.

Boltzmann and Klemencic have both shown that the true dielectric constant of air at a temperature of 0° C. and 760 mm. is 1.00059. That of oxygen gas at the same temperature and pressure is not very different. If the value of $K-1$ for gases varies directly as the pressure, and if temperature *per se* makes no difference, then the dielectric constant of the gaseous oxygen lying on the surface of the liquid oxygen, and which has a temperature of -182° C. nearly, and a density about three times that of the gas at 15° C., is not far from 1.002. Hence the correcting factor to be applied to the above value of the dielectric constant of the liquid is at the most 1.002, and the true dielectric constant of liquid oxygen at -182° C. and under a pressure of 760 mm. is not far from 1.493.

We intend to examine this correction more closely.

As a matter of fact, we were not able to detect any difference between the capacity of the small condenser when held in air at ordinary temperature (15° C.) and pressure, and in the cold gaseous oxygen at -182° C. lying on the surface of the liquid oxygen.

Until we are able to make a better determination we may take the above number, 1.491, therefore, as representing in all probability a close approximation to the dielectric constant of liquid oxygen.

The interesting question then arises how far does liquid oxygen obey Maxwell's law, by which the product of the dielectric constant and the magnetic permeability should be equal to the square of the refractive index for waves of infinite wave-length? The materials are at hand for making this comparison, as we have ourselves just determined the magnetic permeability of liquid oxygen, and find it to be 1.00287,* and the refractive index of liquid oxygen has been determined by Professors Liveing and Dewar for several different wave-lengths.†

* See Fleming and Dewar, 'Roy. Soc. Proc.,' December, 1896, vol. 60, p. 283, "On the Magnetic Permeability of Liquid Oxygen and Liquid Air."

† Liveing and Dewar, 'Phil. Mag.,' Sept., 1895, p. 269, "On the Refraction and Dispersion of Liquid Oxygen and the Absorption Spectrum of Liquid Air." See also Liveing and Dewar "On the Refractive Index of Liquid Oxygen," 'Phil. Mag.,' August, 1892, "On the Spectrum of Liquid Oxygen and on the Refractive Indices of Liquid Nitrous Oxide and Ethylene;" also Liveing and Dewar, 'Phil.

Professors Liveing and Dewar determined the refractive indices (μ) corresponding to certain wave-lengths (λ) for the following wave-lengths :—

From lines in the spectrum of	λ .	μ .
Cadmium	4416	corresponds to 1·2249
	6438	„ 1·2211
Thallium.....	5350	„ 1·2219
Lithium.....	6705	„ 1·2210
Sodium	5892	„ 1·2114

They state that they consider the best results are given by the first two observations. Taking these wave-lengths 4416 and 6438, and the refractive indices corresponding to them, we have calculated from them, by the formula

$$\mu_{\infty} = \frac{\mu\lambda^2 - \mu_1\lambda_1^2}{\lambda^2 - \lambda_1^2},$$

the refractive index for infinite wave-length (μ_{∞}) and found it to be as follows :—

$$\mu_{\infty} = 1\cdot2181.$$

The square of this number is 1·4837, and this, therefore, is the value of the square of the refractive index for waves of infinite wave-length in liquid oxygen.

Taking the product of the dielectric constant, $K = 1\cdot491$, as above determined, and that of the magnetic permeability, $p = 1\cdot00287$, as previously obtained by us, we find that this product Kp is 1·495, and hence that there is therefore a very fairly close agreement between the number representing the square of the refractive index for waves of infinite wave-length and the above product. The difference amounts to about two-thirds of one per cent. Hence liquid oxygen is a substance which very closely obeys Maxwell's law.

We have applied the same apparatus to the determination of the dielectric constant of liquid air obtained in exactly the same manner, and Table II below gives the results of the observations taken in liquid air. The observed results, when corrected as above described, give for the dielectric constant of liquid air the number 1·495, which is slightly more than that of the liquid oxygen. As, however, by the time the experiment was complete the liquid air had practically become liquid oxygen owing to the nearly complete evaporation of the nitrogen, the coincidence of the two results is only what was to be expected.

Mag.,' October, 1893, "On the Refractive Indices of Liquid Nitrogen and Air;" also Liveing and Dewar, 'Phil. Mag.,' Sept., 1888, "On the Absorption Spectrum (luminous and ultra-violet) of large Masses of Oxygen."

The Table II below gives the observational results in the case of the liquid air—really, however, of liquid oxygen.

Table II.—Dielectric Constant of Liquid Air (practically Liquid Oxygen).

In Liquid Air. Ballistic throw for condenser charged to 100 volts.	In cold Gaseous Air. Ballistic throw for condenser charged to 100 volts.
9.5	—
9.6	—
9.5	—
—	6.5
—	6.6
—	6.6
9.4	—
9.5	—
9.55	—
—	6.51
—	6.51
9.7	—
9.55	—
Mean = 9.54	Mean = 6.54
Dielectric constant = 1.495.	

With regard to the above-determined dielectric constants for liquid oxygen and liquid air, it may be remarked that these numbers are smaller than those which have been obtained for almost any other solid or liquid substance of which we have been able to find the measured results. It has been already pointed out that large dielectric constant generally accompanies small specific resistance in a dielectric, and *vice versâ*. Hence, as the specific resistance of the liquid oxygen is very large—it being a very fine insulator—it is not surprising to find the dielectric constant very small. As above mentioned, at a very low temperature, the dielectric constant of some other solid dielectrics has been found by us to be very much reduced, and hence an interesting field of research is opened out for the examination of the change produced by low temperatures on the dielectric constants of other well-known solid insulators, such as paraffin, ebonite, gutta-percha, mica, sulphur, spermaceti, and various frozen liquid insulators, such as the numerous hydrocarbon oils, carbon disulphide, ice, &c.* We hope to

* Mr. W. Cassie, M.A., 'Phil. Trans.,' vol. 46, 1889, has given the results of measurements on the changes produced in the dielectric constants of various insulators by heating them. As far as we can see, our initial results at low temperatures for glass and paraffin are consistent with his. It will be interesting to see how this relatively small dielectric constant of liquid oxygen compares with that of other dielectrics when these last are cooled to the same temperature.

be in a position shortly to furnish further information on this point, and, also, if possible, to say whether the fall in dielectric constant is accompanied by a reduction in the refractive index; that is to say, whether Maxwell's law is obeyed at low temperatures.

We may add that we have already devised a method by which it will be possible to construct a condenser without the above-described distance pieces, and hence to free the resulting measurement from the small uncertainty—amounting, perhaps, to about 1 per cent.—which may affect the above-given numerical results, and which comes in in consequence of the doubt existing as to the exact area of the separators, and also the exact dielectric constant of the glass at the low temperature.

It is interesting to observe that the numbers which we have found above for the dielectric constant of liquid oxygen and liquid air are not very different in order, though somewhat smaller than the dielectric constant as already determined for some other liquid gases,* such as nitrous oxide and carbon dioxide.

In conclusion, we may add that we have been again much indebted to Mr. J. E. Petavel for his kind assistance in making the above-described observations and measurements.

Note added December 15.

In connection with the above investigation, it is interesting to note one remarkable difference between the magnetic susceptibility of oxygen in the liquid and in the gaseous state. The mass of 1 c.c. of gaseous oxygen, taken at 15° C. and 760 mm., is 0.00134 gramme. The mass of 1 c.c. of liquid oxygen, taken at -182° C. and 760 mm., as determined by one of us (J. Dewar), is 1.1375 gramme. Hence the ratio of the density of liquid oxygen to that of gaseous oxygen is 849 to 1.

The magnetic susceptibility of gaseous oxygen at 15° C. and 760 mm., as obtained from the figures given by Faraday and E. Becquerel, is 0.143×10^{-6} per unit of volume, whilst the magnetic susceptibility in the liquid state is, as we have shown,† 228×10^{-6} . Hence the ratio of the magnetic susceptibility of liquid oxygen to that of gaseous oxygen for equal volumes is 1594 to 1.

In other words, the magnetic susceptibility of liquid oxygen is nearly twice as great as that of gaseous oxygen for equal masses. The inference is that magnetic susceptibility is not merely a property of the molecule *per se*, but is a function of the state of aggregation.

* See F. Linde, 'Journal de Physique,' vol. 5, Sept., 1896, p. 413, "On the Dielectric Constant of Liquid Gases."

† See Fleming and Dewar, 'Roy. Soc. Proc.,' vol. 60, p. 283, December, 1896.

Note added December 18.

In addition to the arrangements above described for determining the capacity of the small condenser, we have also employed the well-known method of charging and discharging the small condenser through a galvanometer by means of a contact-maker driven at a speed of sixty contacts per second by an electrically controlled tuning-fork. By this means a steady deflection of the galvanometer is obtained due to the passage of the rapidly recurring discharges through it. Preliminary observations with this apparatus have confirmed the above-given value for the dielectric constant of liquid oxygen, and by a modification of it we hope shortly to make a very careful re-determination of the constant.

“On Subjective Colour Phenomena attending sudden Changes of Illumination.” By SHELFORD BIDWELL, M.A., LL.B., F.R.S. Received December 10,—Read December 17, 1896.

The investigation which forms the subject of this paper originated in an attempt to account satisfactorily for the colour phenomena exhibited by Mr. C. E. Benham’s “Artificial Spectrum Top,” which, when it was brought before the public, about two years ago, excited considerable interest.

The top consists of a disk of cardboard about $4\frac{1}{4}$ in. (10·8 cm.) in diameter, mounted upon a spindle. One half of the disk is painted black; upon the white ground of the other half are drawn four successive groups of three black lines, having the form of concentric arcs of 45° , which are at different distances from the centre, as shown in the annexed figure; the thickness of the lines is about $\frac{1}{25}$ in. (1 mm.). When the disk rotates, each group of black lines generally appears to assume a different colour.

The nature of the colours thus developed depends upon the speed of the rotation, and upon the quality and intensity of the illumination. After several trials, I found that no better results, on the whole, could be obtained than when the disk was illuminated by a 16-candle power incandescent lamp, with a ground glass bulb, at a distance of about 6 in. (15 cm.), and was caused to turn about five times in a second. These, therefore, were adopted as the standard conditions for my experiments, the disk being mounted upon a horizontal axis, driven by an electro-motor, and the speed regulated by comparison with the ticks of an ordinary watch.

When the disk rotates under the specified conditions and in the direction indicated by the arrow in the figure, the inner group of